

EXPERIMENTAL STUDY OF DAMPING COEFFICIENT OF  
DASH-POT DAMPERS

A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of

MASTER OF TECHNOLOGY

by

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CERTIFICATE

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## LIST OF SYMBOLS

$A$	First integration constant.
$A_a$	Area of annulus.
$B_x, B_y, B_z$	Body forces along x, y and z directions, respectively.
$b$	Width of the beam.
$C$	Coefficient of damping.
$C_c$	Critical damping.
$C_e$	Coefficient of damping for eccentricity e.
$C_o$	Coefficient of damping for concentric case.
$C_r$	Radial clearance.
$C_{r\theta}$	Radial clearance at an angle $\theta$ .
$C_s$	Experimental coefficient of damping before correction.
$C_{exp}$	Experimental coefficient of damping
$(C_{exp})_o$	Experimental coefficient of damping for concentric case.
$(C_{exp})_e$	Experimental coefficient of damping for eccentric case.
$(C_{exp})_{df}$	Experimental coefficient of damping for particular damping fluid.



$C_{emp}$	Coefficient of damping from empirical relation
$(C_{emp})_o$	Coefficient of damping from empirical relation for concentric case
$(C_{emp})_e$	Coefficient of damping from proposed empirical relation for eccentric case
$(C_{emp})_{df}$	Coefficient of damping from proposed empirical relation for particular damping fluid
$C_{the}$	Coefficient of damping from theory
$(C_{the})_o$	Coefficient of damping from theory for concentric case
$D_c$	Cylinder diameter
$D_p$	Piston diameter
$D_R$	Piston rod diameter
$d$	Depth of beam
$E$	Modulus of elasticity of material of the beam
$e$	Eccentricity
$F$	Force of resistance
$f$	Frequency in cycles
$f_o$	Undamped natural frequency
$F_v$	Viscous resistance force
$g$	Acceleration due to gravity
$H$	Height of piston
$h$	Depth of immersion
$I$	Moment of inertia of the beam
$K$	Difference, inverses of square of $r_1$ and $r_2$
$L$	Length of the beam
$M$	Mass attached at the center of the beam
$M_{eq}$	Equivalent mass of the system

$m$	Mass per unit length of the beam.
$N$	Wire constant.
$n$	Number of cycles.
$p$	Pressure.
$q$	Velocity of the fluid.
$r_1$	Outer radius of plunger.
$r_2$	Inner radius of smaller cup
$T$	Time period in seconds.
$t$	Time.
$U$	Piston velocity.
$u$	Velocity of fluid element in x-direction.
$u_{avg}$	Average velocity.
$u_{avg,\theta}$	Average velocity at an angle $\theta$ .
$v$	Velocity of fluid element in y-direction.
$w$	Velocity of fluid element in z-direction.
$x$	x co-ordinate variable.
$x_n$	Magnitude of $n^{th}$ peak of amplitude.
$y$	y co-ordinate variable.
$z$	z co-ordinate variable.
$\beta$	Pressure difference multiplier.
$\delta$	Logarithmic decrement.
$\Delta f$	Difference of frequencies.
$\Delta x, \Delta y, \Delta z$	Size of the fluid element
$\theta$	Angle

**x**

$\mu$	Coefficient of viscosity of damping fluid,
$\nu$	Kinematic viscosity.
$\xi$	Damping ratio ; $C/C_c$
$\rho$	Density of damping fluid.
$\tau_{px}$	Shear stress at piston wall.
$\tau_{yx}$	Shear stress.
$\tau_{wx}$	Shear stress at cylinder wall
$\omega$	Speed of angular rotation.
$\omega_n$	Natural frequency in radians .per second.

ABSTRACT

Present work is an experimental investigation of the theory of coefficient of damping of dash-pots with large clearances, for vibrating systems. Experiments indicate large difference between the theoretical and experimental values especially for smaller clearances, contrary to the expectations. An empirical formula is proposed to fit in the experimental results.

Existing theory for eccentric case has been extended and improved to predict the value of coefficient of damping for any eccentricity. This has been verified experimentally.

## CHAPTER - 1

### INTRODUCTION

Fatigue failure, wear and undue generation of noise of mechanical systems are many times due to excessive vibrations of the systems. To overcome such problems various damping devices are used, which dissipate the excessive energy of the system and thus reduce the undesirable vibrations. One of the commonly used devices is hydro-mechanical damping in which damping occurs through the fluid flow. If in this fluid friction is utilized to dissipate the energy of the mechanical systems then such damping device is generally called dash-pot damper. Such dampers are mainly used to

- (i) resist that is stop or retard an inertial load
- (ii) resist an applied force
- (iii) absorb shock or vibrations.

Irrespective of the function, basically dash-pot in its most simple form consists of a piston or plunger moving in a cylinder, filled with viscous damping fluid whose viscosity acts as a source of energy loss as and when required by the system.

To get the desired amount of damping one has to choose dimensions of the dash-pot and suitable damping fluid. Such characteristics of dash-pot dampers have been the subject of number of investigations.

It seems it was Peterson (8) who for the first time tried to answer this problem. Since then number of papers have been written on the dash-pot theory, and all yield essentially the same results. Peterson reported some experimental results on dash-pots with small radial clearances. As far as the author is aware, no comparable data for dash-pots with large clearances is available. The present work reports some measurements on such dash-pots.

Old work is reviewed in the Sec. 1.1. Chapter-2 presents the usual theory for concentric dash-pots for the sake of continuity and its extension for any eccentricity. Chapter 3 gives the design of experimental set-up to verify the theory. Chapter 4 gives the results of the experiments. In Chapter 5 theoretical and experimental results are discussed and empirical relation has been proposed in the light of experimental results.

## 1.1 REVIEW OF OLD WORK :

First analytical relation for the design of dash-pots was presented by Peterson (8). He developed mathematical equations for the velocities of piston as a function of applied force and geometry of the dash-pot. It is observed from (8) that coefficient of damping is inversely proportional to the cube of the radial clearance of the damper and linearly proportional to the height of the piston, cube of diameter of piston and viscosity of the damping medium. Further, he developed equations for eccentric movement of piston in the cylinder for the maximum eccentricity, and showed that the above proportionalities also hold for this case. He also made an attempt to verify these relations experimentally. For a given force he measured velocities of the pistons for extreme eccentricity conditions only. Moderate agreement was found for small Reynold Numbers,  $Re$ . For large  $Re$ , turbulent flow exists, and the experimental velocities were found to be much lower than those calculated on the assumption of laminar flow.

Ref. (1) comes up with same conclusion for linearly moving dash-pots as Peterson (8) but derived in different way. No experimental work was performed. So, it was nearly a repeat of Peterson's (8) theoretical work

In the same year above work (1) was supplemented by Popper(9). He suggested corrections for piston velocity, velocity gradient near the piston wall and inertia forces. Applying these corrections he found the damping coefficient to be 22% more than the previous theory for a damper having ratio of radial clearance to piston dia as 0.15. Though Popper(9) made good improvements in then existing theory<sup>(1,8)</sup>, but as a whole it took the value of damping coefficient still farther from the experimental ones.

After this paper basically no improvements in the theory has been made. Dodge(2) gives the expression for pressure difference causing fluid to flow in dash-pots. Of course when this relation is modified for damping coefficient, it turns out to be the same as obtained in Ref. (8, 1). Apart from this he studied many types of orifices and flow conditions which do not fall within the field of viscous dampers. Again in the same year Dodge(3) derives expressions for stopping time of plunger in case of viscous dampers using the energy balance equation where he finds the stopping time to be inversely proportional to the cube of the radial clearance. He derives expressions for a complicated geometrical configuration of dash-pot which when simplified for the simple dash-pot turns out to be almost same as previous work (1, 2, 8),



that is, that coefficient of damping is proportional to the cube of the ratio of mean of diameters of piston and cylinder to the radial clearance.

Grover (4) discusses the design and performance of a variable dash-pot. He performed the experiments using two slotted pistons one above the other moving in the cylinder filled with damping fluid and obtained damping coefficients for different overlappings of the slots of the pistons. He does not give any analytical relations to support his work.

## CHAPTER - 2

### THEORY FOR DAMPING FACTOR

In its simplest form dash-pot consists of a piston operating in a cylinder with a small clearance between piston and cylinder walls as shown in Fig. 1. When the load is applied to the piston, the viscous fluid trapped below the piston is forced through the annular clearance between cylinder and piston. Moving piston displaces the fluid, causing pressure rise which drives the fluid through the annular space. For the development of the theory (1,8) following idealisations are made :

- (i) Damping medium is a viscous fluid and incompressible.
- (ii) Radial clearance,  $C_r$ , between cylinder and piston is small in comparison to the diameter of the piston.
- (iii) Piston height,  $H$ , is sufficient to ensure the fluid flow to be fully developed that is sharp edge effects are avoided.
- (iv) Fluid flow is laminar.

- (v) Piston rod diameter is very small as compared to piston diameter.
- (vi) Free area above the piston is large enough so that the fluid velocity in this area approaches zero value.
- (vii) Speed of the piston and radial clearance is limited to such a value so as not to cause cavitation effect due to large pressure drop.
- (viii) Fluid inertia forces are negligible as compared to the load and piston inertia.

Theory for damping factors is being given for the following two cases

- (a) a circular piston travelling coaxially in a circular cylinder based on Ref. (1,8)
- (b) a circular piston travelling eccentrically in a circular cylinder. (Extension of Ref. (1,8))

## 2.1 CIRCULAR PISTON TRAVELLING COAXIALLY IN A CIRCULAR CYLINDER :

As shown in Fig. 1, piston is moving down with speed  $U$ . Viscous fluid trapped below the piston is forced through the annular space in the positive  $x$ -direction.

A co-ordinate system is chosen as shown in the figure. Flow of the damping fluid is governed by the continuity equation and Navier Stokes equations (13)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = B_x - \frac{1}{\rho} \frac{\partial p}{\partial x} \\ + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = B_y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\ + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = B_z - \frac{1}{\rho} \frac{\partial p}{\partial z} \\ + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{aligned} \quad (4)$$

where  $u$ ,  $v$  and  $w$  are the velocity of the fluid element in  $x$ -,  $y$ - and  $z$ -directions, respectively. As the fluid motion is predominantly in  $x$ -direction so velocities  $v$  and  $w$  are considered to be negligible, that is,  $v=w=0$ . Therefore continuity equation (1) becomes

$$\frac{\partial u}{\partial x} = 0 \quad (5)$$

As it has been idealised that radial clearance is very small compared to the circumference of the piston, fluid can be considered as flowing between two parallel plates with the upper plate moving with a velocity  $U$  in negative x-direction as shown in Fig. 2. Due to symmetry, velocity of fluid elements at all points on the circumference of the piston will be same

$$\text{that is } \frac{\partial u}{\partial z} = 0$$

And also neglecting inertia and body forces Navier-Stokes equations (2,3,4) reduce to

$$\frac{1}{\rho} \frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\frac{\partial p}{\partial y} = 0 \quad (7)$$

$$\frac{\partial p}{\partial z} = 0 \quad (8)$$

Equations (6,7,8) imply that  $p$  is a function of  $x$  only. From Eqn. (5)  $u$  can be function of  $y$  and  $z$ . But since  $\frac{\partial u}{\partial z} = 0$ , therefore  $u$  can be function of  $y$  only that is  $u = u(y)$ .

Therefore Eqn. (6) can be written in the form of total derivatives that is,

$$\frac{dp}{dx} = \nu \frac{d^2 u}{dy^2} \quad (9)$$

Since  $u$  is a function of  $y$  only, therefore the left hand side of Eqn. (9) can be either a function of  $y$  or a constant. As explained earlier pressure,  $p$ , can be a function of  $x$  only, so it follows that both sides of Eqn. (9) must be equal to a constant.

Integrating Eqn. (9), one gets

$$u = \frac{1}{\mu} \frac{dp}{dx} \frac{y^2}{2} + A y + B \quad (10)$$

Boundary conditions for the flow are

at  $y = 0$  ,  $u = 0$

and at  $y = C_r$  ,  $u = -U$

With these boundary conditions Eqn. (10) becomes

$$u = -\frac{U}{C_r} y + \frac{1}{2\mu} \frac{dp}{dx} (y^2 - C_r y) \quad (11)$$

Velocity profile  $u$  is not fully determined as the value of  $dp/dx$  is not known.  $dp/dx$  can be determined by using the fact that the rate at which fluid passes through the annulus space must be equal to that of the fluid displaced by piston, that is,

$$\frac{\pi}{4} D_p^2 U = \pi \int_0^{C_r} (D_c - 2y) u dy \quad (12)$$

Plugging value of  $u$  from Eqn. (11) into Eqn. (12), one gets

$$\frac{dp}{dx} = - \frac{3 \mu U}{C_r^3} \beta \quad (13)$$

where 
$$\beta = \left( \frac{(D_c - 2 C_r)^2}{(D_c - C_r)} + 2 C_r \frac{(D_c - \frac{4}{3} C_r)}{(D_c - C_r)} \right) \quad (14)$$

Integration of Eqn. (13) gives the pressure difference between the bottom side and top side of piston. This pressure difference causes damping fluid to flow through the annulus. Resistance force,  $F$ , can be determined as

$$F = \left( - \int_0^H (dp/dx) \times A \right) \quad (15)$$

where  $A$  is the area of piston on which total pressure difference is acting. On substituting  $dp/dx$  from Eqn. (13), Eqn. (15) becomes

$$F = \frac{3 \pi \mu U H \beta (D_c - 2 C_r)^2}{4 C_r^3} \quad (16)$$

Dividing the Eqn. (16) by  $U$ , the coefficient of viscous damping,  $C$ , is determined as follows:

$$C = \frac{F}{U} = \frac{3 \pi \mu}{4 C_r^3} H \left( \beta (D_c - 2 C_r)^2 \right) \quad (17)$$

On substituting value of  $\beta$  from Eqn. (14), value of  $C$  becomes

$$C = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2 C_r}{C_r} \right)^3 \left( \frac{D_c - 2 C_r}{D_c - C_r} + \frac{2 C_r}{D_c - 2 C_r} \cdot \frac{D_c - \frac{4}{3} C_r}{D_c - C_r} \right)$$

As  $C_r \ll D_c$  ;  $\therefore C = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2 C_r}{C_r} \right)^3$  (18)

Thus coefficient of viscous damping is linearly proportional to the viscosity of damping fluid, height of piston, and is cubic function of the ratio of cylinder diameter to radial clearance.

## 2.2 PISTON MOVING ECCENTRICALLY IN THE CYLINDER :

Referring to Fig. 3, let A be the centre of the piston and D that of the cylinder. Here again  $v = w = 0$  is assumed to be a good approximation, since  $C_r \ll D_c$ .

$$\text{Now } AB = \frac{D_p}{2} ; \quad DB = D \text{ and } DC = \frac{D_c}{2} \quad (19)$$

$$\text{Eccentricity} = AD = e$$

Let  $C_{r\theta} = BC$  = radial clearance at any angle  $\theta$  where  $\theta$  is the angle which DBC makes with the centre line ADE. BE is perpendicular to ADE. Angle  $\theta$  is to be taken positive in anticlockwise direction.

From  $\triangle ABE$ ,

$$AB^2 = (AD + DE)^2 + BE^2 \quad (20)$$

Substituting values from Eqn. (19) and rearranging Eqn.

(20) becomes

$$D^2 + 2 (e \cos \theta) D + e^2 - \frac{D_p^2}{4} = 0$$



Solving for D, we get

$$D = -e \cos \theta \pm \sqrt{\frac{D_p^2}{4} - e^2 \sin^2 \theta} \quad (21)$$

$$\text{Now } C_{r\theta} = BC = DC - DB \quad (22)$$

Substituting the values from Eqns. (19) and (21) in Eqn. (22),

$$C_{r\theta} = \frac{D_c}{2} + e \cos \theta \mp \frac{D_p}{2} \sqrt{1 - \frac{4}{D_p^2} e^2 \sin^2 \theta}$$

For all practical purposes  $\frac{4 e^2 \sin^2 \theta}{D_p^2} \ll 1$

$$\text{Therefore } C_{r\theta} = \frac{D_c}{2} + e \cos \theta - \frac{D_p}{2}$$

or

$$C_{r\theta} = C_r + e \cos \theta \quad (23)$$

Now for a small element lmno at angle  $\theta$  flow conditions are at  $y = 0, u = 0$

and at  $y = C_r + e \cos \theta, u = -U$

Using above boundary conditions in Eqn (10), one gets

$$u = -\frac{U}{C_r + e \cos \theta} y + \frac{1}{2\mu} \frac{dp}{dx} \{y^2 - (C_r + e \cos \theta)y\} \dots \quad (24)$$

Let  $u_{avg,\theta}$  be the average velocity of fluid through this element, then

$$u_{avg,\theta} = \frac{1}{C_r + e \cos \theta} \int_0^{C_r + e \cos \theta} u \, dy \quad (25)$$

The fluid displaced by the piston must be equal to the fluid passing through the passage between piston and cylinder, that is,

$$\frac{\pi}{4} D_p^2 U = \frac{\pi}{4} (D_c^2 - D_p^2) u_{avg} \quad (26)$$

where

$$u_{avg} = \frac{1}{2\pi} \int_0^{2\pi} u_{avg,\theta} d\theta \quad (27)$$

Substituting value of  $u_{avg,\theta}$ , Eqn. (26) becomes

$$\begin{aligned} \frac{\pi}{4} D_p^2 U &= \frac{\pi}{4} (D_c^2 - D_p^2) \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{1}{C_r + e \cos \theta} \int_0^{C_r + e \cos \theta} \left( -\frac{U}{C_r + e \cos \theta} y + \frac{1}{2\mu} \frac{dp}{dx} (y^2 - (C_r + e \cos \theta)y) \right) dy \right) d\theta \quad (28) \end{aligned}$$

Integrating Eqn. (28) and rearranging, one gets (For details see Appendix A

$$\frac{dp}{dx} = - \frac{12\mu (D_p^2 + (D_p + D_c) C_r) U}{(D_p + D_c) C_r (2 C_r^2 + e^2)} \quad (29)$$

As in Sec. 2.1, coefficient of damping is found as

$$C_e = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2 C_r}{C_r} \right)^3 \left( \frac{D_c - 2 C_r}{D_c - C_r} + \frac{2 C_r}{D_c - 2 C_r} \right) / \left( 1 + \frac{1}{2} \left( \frac{e}{C_r} \right)^2 \right)$$

As  $C_r \ll D_c$ , this may be written as

$$C_e = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2 C_r}{C_r} \right)^3 / \left( 1 + \frac{1}{2} \left( \frac{e}{C_r} \right)^2 \right) \quad (30)$$

Eqn. (30) implies that eccentric dash-pot system has lower coefficient of damping than that for concentric dash-pots. Difference between this analysis and of Ref. (8) is discussed in Chapter 5.

## CHAPTER - 3

### EXPERIMENTAL VERIFICATION OF THEORY OF DASH-POTS

#### 3.1 METHODS OF MEASURING DAMPING :

It was desired to measure the damping coefficient while using the dash-pot as a part of a vibrating system. This ruled out the use of Peterson's (8) method, which consisted in measuring the velocity for a given force. The following methods were considered for measuring the damping coefficient:

- 3.1.1 Amplification factor method.
- 3.1.2 Band width method.
- 3.1.3 Logarithmic decrement method.

##### 3.1.1 Amplification Factor Method:

In a single degree of freedom system, if a constant excitation force is applied with gradually increasing frequency, it is found that the amplitude of vibration steadily increases to a maximum and then decreases as the frequency is further increased. So there is one value of frequency at which the amplitude would be maximum and this amplitude is a measure of damping coefficient of the system.

### 3.1.2 Band Width Method:

This method is based upon the difference in the two frequencies at which the amplitude is the same if the exciting force is the same. For a linear viscously damped system damping ratio  $\xi$  can be determined as follows (10) :

$$2\xi = \frac{\Delta f}{f_0} \sqrt{\frac{x^2}{x_{\max}^2 - x^2}} \quad (31)$$

where  $\Delta f$  is the difference between the two frequencies at which amplitude is  $x$  and  $f_0$  is the undamped natural frequency.  $x_{\max}$  is the amplitude at  $f_0$ . But commonly used criterion is

$$x_{\max} = \sqrt{2} x$$

then eqn. (31) becomes

$$\xi = \frac{\Delta f}{2 f_0} \quad (32)$$

### 3.1.3 Logarithmic Decrement Method:

Experimental procedure is to let the system vibrate freely and obtain a record of the response. This response is as shown in Figure 4. From it system damping is computed as follows (11)

$$\delta = \frac{1}{n} \ln \frac{x_0}{x_n} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad (33)$$

where  $n$  is the number of cycles between which amplitude reduces from  $x_0$  to  $x_n$  and  $\xi$  is the ratio of coefficient of damping to critical damping. Knowing the critical damping of system, coefficient of damping,  $C$ , can be determined.

### 3.2 COMPARISON OF EXPERIMENTAL TECHNIQUES OF MEASURING DAMPING COEFFICIENT :

Method of amplification factor measurements is difficult to use for absolute measurement of damping, since reference level is hard to find, and moreover this method does not give good results in case of large dampings. Band width method is a widely used measurement technique, which needs force of excitation to be maintained constant, and then varying the frequency, damping ratio is determined as already explained.  $\Delta f$  has to be kept as small as possible to minimize interference from other modes of specimen or support structure. If  $\Delta f$  is made small, then results become more error prone. Moreover errors are likely to increase, as damping ratio increases. In the above two methods response diagram is needed, so to determine one value of  $C$  one has to take large number of readings, which make experiments to be very time consuming. Whereas on the other hand logarithmic decrement method is quite simple to use as no excitation is needed. In this method value of  $C$  is determined from the free

vibration amplitude record taken with the help of suitable transducer. This method is very fast, as only one observation is needed for finding value of  $C$ . So this method is best and simple to use except that it can not be used for higher modes.

### 3.3 DESIGN OF EXPERIMENTAL SET UP:

Accurate experimental determination of coefficient of damping mainly depends upon the degree of accuracy of damping ratio. So, for this the experimental set up should give sufficient number of cycles of vibrations before its amplitude approaches zero. Thus critical damping of the system should be as large as possible and variable to accommodate large number of experiments on different dash-pots. Moreover set up needs ease of determination of its critical damping. One such system is beam. Cantilever was ruled out as piston of the dash-pot will not run co-axially in the cylinder. So clamped-clamped beam was chosen. Clamping supports were so designed as to accept different lengths, widths and heights of the beam to facilitate the variation of the critical damping. This can also be changed by attaching additional mass at the centre of the beam. So, for this purpose piston rod attached at the centre of the beam, was not only designed for transmitting free vibrations of the beam to the dash-pot but also its upper portion was threaded to hold firmly additional masses.

Then the next requirement of the set up is the proper travel of piston that is piston should run straight

and parallel to the axis of the cylinder with required amount of eccentricity (including  $e = 0$ ). Straight and parallel motion of the piston was ensured by attaching the piston rod at the centre of the beam. The required amount of eccentricity was obtained by using two micrometer screw unislides fixed at right angles to each other. Amount of eccentricity was easily determinable as pitches of micrometer screws were known. To bring the piston at the centre of the cylinder, use of electric circuit continuity was made, which used to glow a bulb when piston touched the side of the cylinder wall. Then readings of both glowings along any diameter were taken and then cylinder was brought to the central position. Similarly it was set for the other diameter at right angles to the previous one, making piston to be concentric within accuracy of 0.05 mm. And for desired eccentricity cylinder was moved along any diameter using any of the micrometer screw unislides.

*of theory (1, 8)*

The verification/needs determination of damping ratio from the logarithmic decrement of free vibration of beam. For which the displacement curve of the system was obtained with the help of the electric resistance strain gages. Its signal was fed to Budd's strain indicator and then to Visicorder. Permanent record of the motion of

beam was obtained on the light sensitive paper, written by a beam of light in the Visicorder recorder. To further improve the accuracy of the logarithmic decrement, microscope with high magnification power was used to measure amplitude lengths of the displacement record.\*

Complete experimental set up is shown in Fig.5.

### 3.4 EXPERIMENTAL DETERMINATION OF DAMPING COEFFICIENT:

For small values of  $\xi$ , coefficient of damping,  $C$ , can be determined from Eqn. (33) as follow:

$$C = \frac{\xi}{\omega_n} \text{ Meq } \omega_n \quad (34)$$

where  $\omega_n$  is natural frequency of beam and Meq is the equivalent mass and are given by Timoshenko (12)

$$\omega_n = \sqrt{\left(\frac{192 E I}{L^3 \text{ Meq}}\right)} \quad (35)$$

$$\text{Meq} = \frac{192}{504} m L + M \quad (36)$$

and

$E$  = Young modulus of the material of beam

$I$  = Moment of inertia =  $\frac{1}{12} b d^3$

$b$  = width of the beam

$d$  = thickness of the beam

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\* The possibility of cavitation during the upstroke is discussed in Chapter 5.



$L$  = Length of the beam

$M$  = concentrated mass attached at the centre  
of the beam

$m$  = mass per unit length of beam

To compare the above experimental values of coefficient of damping of dash-pot with the theoretical values, value of the coefficient of viscosity at the time (temperature) of experimentation is needed. Viscosity of the damping fluid was measured from time to time experimentally as given in Appendix B.

For successful verification following precautions were observed:

- (i) Before filling the damping fluid in the cylinder, the level of the cylinder top was checked for horizontal level with the help of spirit level of 0.02 mm/m
- (ii) While filling the oil into the cylinder it was ensured that no air bubbles were trapped within the fluid.
- (iii) Proper care for heating up of the viscidorder recorder was taken up to stablize the instrument to give repetitive values.

- (iv) Depending upon temperature change during experimentation viscosity of damping fluid was measured time to time.
- (v) For a chosen set of parameters, observations were repeated atleast twice. Good repeatability was observed.

## CHAPTER - 4

### EXPERIMENTAL RESULTS

Using the experimental set up as described in Chapter-3, experiments were planned as given below to verify the theory described in Chapter-2. Sample calculations are given in Appendix C

#### 4.1 EFFECT OF RADIAL CLEARANCE ON COEFFICIENT OF DAMPING FOR CONCENTRIC DASH-POTS:

(a) Dash-pot,  $D_c = 70.94 \text{ mm}$

Experiments 1.1 to 1.5 were performed with piston height as 8 mm with different critical dampings. Experiments 1.6 to 1.7 were on 12 mm piston height with different critical dampings.

(b) Dash-pot,  $D_c = 50.00 \text{ mm}$

Experiment 1.8 and 1.9 were done with 5 mm piston height with different critical dampings. And experiments 1.10 and 1.11 were on pistons with height 12 mm.

In the above experiments for a particular set up viscosity was maintained same and different sizes of pistons were used to get different radial clearances.

#### 4.2 EFFECT OF PISTON HEIGHT ON COEFFICIENT OF DAMPING:

(a) Dash-pot,  $D_c = 70.94$  mm

Experiment 2.1 was performed with piston diameter 65 mm.

(b) Dash-pot,  $D_c = 50.00$  mm

Experiment 2.2 was performed with piston diameters 47.2 and 48 mm respectively, and experiment 2.3 was performed with piston diameter 46.4 mm.

#### 4.3 EFFECT OF VISCOSITY OF DAMPING FLUID ON COEFFICIENT OF DAMPING :

(a) Dash-pot,  $D_c = 70.94$  mm

Experiment 3.1 was performed with different damping fluids with piston diameter 65.8 mm and 8 mm height.

(b) Dash-pot,  $D_c = 50.00$  mm

Experiments 3.2, 3.3 and 3.4 were performed with piston diameters 45.6, 47.2 and 44 mm respectively with different damping fluids.

#### 4.4 EFFECT OF ECCENTRICITY ON COEFFICIENT OF DAMPING:

(a) Dash-pot,  $D_c = 70.94$  mm

Experiment 4.1 was performed with base clearance 2.97 mm.

(b) Dash-pot,  $D_c = 50.00$  mm

Experiment 4.2 and 4.3 were performed with base clearance of 1.4 mm and 3 mm respectively.

Here base clearance means radial clearance when piston and cylinder are concentric.

Results for these experiments are given in corresponding Tables. Also results of experiments 1.1 to 1.11 are given in Fig. 6 and results of other experiments in corresponding figures.

Error estimates for the above results are given in Appendix D.

TABLE NO. 1.1

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Damping fluid - Usha Machine Oil

Temperature = 30.5°C

Viscosity of oil = 0.2487 N - s / m<sup>2</sup>

Mass equivalent of the system = 3.859 Kg

Natural frequency of the system = 95.0 rad/s.

Experimental Correction for C = 0.582 N - s / m

Cylinder diameter = 70.94 mm, piston height = 8 mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp</sub> N - S/m	C <sub>the (1,8)</sub> N - S/m
1	13.47	0.1898	0.0072	0.815	0.233	0.360	0.163
2	10.67	0.1504	0.0128	1.490	0.908	0.629	0.238
3	8.22	0.1158	0.0157	1.828	1.246	1.122	1.365
4	7.17	0.1010	0.0250	2.911	2.329	1.513	2.305
5	5.47	0.0771	0.0368	4.287	3.705	2.716	6.173
6	4.97	0.0700	0.0438	4.636	4.054	3.333	8.668
7	4.47	0.0630	0.0427	4.972	4.390	4.151	12.485
8	3.97	0.0559	0.0438	5.348	4.766	5.363	18.751
9	3.17	0.0446	0.0630	7.336	6.754	8.596	39.803
10	2.97	0.0418	0.0812	9.452	8.870	9.847	50.398
11	2.57	0.0362	0.1032	12.920	12.338	13.295	78.669
12	2.17	0.0306	0.1376	16.024	15.442	18.854	133.871
13	1.77	0.0249	0.2430	28.249	27.717	28.653	259.848

TABLE NO. 1.2

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Damping fluid - Usha Machine Oil Temperature = 30.5°C

Viscosity of oil = 0.2487 N - S / m<sup>2</sup>

Mass equivalent of the system = 7.041 Kg.

Natural frequency of the system = 70.164 rad / S

Experimental correction for C = 1.0246 N - S / m

Cylinder diameter = 70.94 mm, piston height = 8mm

S No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp</sub> N - S/m	C <sub>the (1.8)</sub> N - S/m
1	13.47	0.1898	0.0085	1.336	0.311	0.360	0.162
2	10.67	0.1504	0.0072	1.132	0.107	0.629	0.238
3	8.22	0.1158	0.0180	2.838	1.807	1.122	1.365
4	7.17	0.1010	0.0184	2.893	1.868	1.553	2.305
5	5.47	0.0771	0.0265	4.173	3.148	2.716	6.173
6	4.97	0.0700	0.0272	4.272	3.248	3.333	8.668
7	4.47	0.0630	0.0307	4.827	3.803	4.152	12.485
8	3.97	0.0559	0.0364	5.724	4.699	5.363	18.751
9	3.17	0.0446	0.0406	6.384	5.360	8.596	39.803
10	2.97	0.0418	0.0541	8.507	7.483	9.847	50.398
11	2.57	0.0362	0.0682	10.725	9.701	13.295	78.669
12	2.17	0.0306	0.1128	17.750	16.725	18.854	133.870
13	1.77	0.0249	0.1787	28.102	27.077	28.653	259.848
14	0.97	0.0136	0.6501	102.235	101.211	97.550	1684.590

TABLE NO. 1.3

Specifications -	Beam dimensions	1003 X 49.2 X 6 mm
Damping fluid -	Usha Machine Oil	Temperature = 30°C
Viscosity of oil		= 0.2487 N - S / m <sup>2</sup>
Equivalent mass of the system		= 2.499 Kg.
Natural frequency of the system		= 117.84 rad / S
Experimental correction for C		= 0.582 N - S / m
Cylinder diameter		= 70.94 mm piston height=8mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp.</sub> N - S/m	C <sub>theor.</sub> N - S/m
1	10.67	0.1504	0.0103	0.967	0.385	0.629	0.238
2	9.22	0.1299	0.0213	2.000	1.418	0.871	0.864
3	8.22	0.1158	0.0265	2.484	1.902	1.122	1.365
4	7.17	0.1010	0.0202	1.889	1.507	1.513	2.305
5	5.47	0.0771	0.0317	2.972	2.390	2.716	6.173
6	4.97	0.0700	0.415	3.765	3.183	3.333	8.668
7	4.47	0.0630	0.0592	5.556	4.974	4.151	12.485
8	3.57	0.0503	0.0600	5.628	5.046	6.705	26.736
9	3.17	0.0446	0.1003	9.409	8.827	8.596	39.803
10	1.77	0.0249	0.2950	27.662	27.080	28.653	259.890
11	0.97	0.0136	0.8940	83.830	83.248	97.550	1684.500



TABLE NO. 1.4

Specifications - Beam dimension 1003 X 49.2 X 6 mm  
 Damping fluid - Usha Machine Oil Temperature = 30°C  
 Viscosity of oil = 0.24847 N - S / m<sup>2</sup>  
 Equivalent mass of the system = 3.268 Kg.  
 Natural frequency of the system = 103.06 rad / S  
 Experimental correction for C = 0.582 N - S/m  
 Cylinder diameter = 70.94 mm piston height = 8mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp.c</sub> N - S/m	C <sub>th=1.8</sub> N - S/m
1	10.67	0.1504	0.0089	0.962	0.380	0.629	0.238
2	9.22	0.1299	0.0164	1.763	1.181	0.871	0.864
3	8.22	0.1158	0.0251	2.695	2.113	1.122	1.365
4	7.17	0.1010	0.0201	2.155	1.573	1.513	2.305
5	6.67	0.0940	0.0204	2.191	1.608	1.771	3.369
6	5.47	0.0771	0.0224	2.408	1.896	2.716	6.173
7	4.97	0.700	0.0289	3.108	2.526	3.333	8.668
8	4.47	0.0630	0.0500	5.395	4.813	4.151	12.485
9	3.57	0.0503	0.0601	6.450	5.868	6.709	26.736
10	3.17	0.0446	0.0827	8.874	8.292	8.596	39.803
11	1.77	0.0249	0.2455	26.321	25.738	28.653	259.898
12	0.97	0.0136	1.0200	109.358	108.776	97.550	1684.590

TABLE NO. 1.5

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Damping fluid - Usha Machine Oil Temperature = 30°C

Viscosity of the oil = 0.248+7 N - S / m<sup>2</sup>

Mass equivalent of the system = 1.756 Kg

Natural frequency of the system = 141.6 rad / s

Experimental correction for C = 0.582 N - S/m

Cylinder diameter = 70.94 mm piston height = 8mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp</sub> N - S/m	C <sub>the</sub> (1,8) N - S/m
1	10.67	0.1504	0.0134	1.065	0.483	0.629	0.238
2	9.22	0.1299	0.0328	2.957	2.375	0.871	0.864
3	8.22	0.1158	0.0358	2.833	2.251	1.122	1.365
4	6.67	0.0940	0.0407	3.228	2.646	1.771	3.369
5	5.47	0.0771	0.0366	2.902	2.320	2.716	6.173
6	4.97	0.0700	0.0419	3.319	2.732	3.333	8.668
7	4.47	0.0630	0.0540	4.275	3.693	4.152	12.485
8	3.57	0.0503	0.0735	5.822	5.240	6.705	26.736
9	3.17	0.0446	0.1185	9.378	8.796	8.596	39.803
10	1.77	0.0249	0.4220	33.400	32.818	28.653	259.898
11	0.97	0.0136	1.0050	79.543	78.961	97.550	1684.590

TABLE NO. 1.6

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Damping fluid - Usha Machine Oil Temperature = 30°C

Viscosity of oil = 0.2487 N - s / m<sup>2</sup>

Mass equivalent of the system = 1.756 Kg.

Natural frequency of system = 140.58 rad / s

Experimental correction for C = 0.582 N - s / m

Cylinder diameter = 70.94 mm Piston height = 12mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	$\delta$	C <sub>d</sub> N - s/m	C <sub>exp</sub> N - s/m	C <sub>emp</sub> N - s/m	C <sub>the (4,8)</sub> N - s/m
1	10.67	0.1504	0.01267	1.304	1.412	0.943	0.357
2	9.22	0.1299	0.01541	2.582	2.000	1.306	1.297
3	8.22	0.1158	0.01604	2.524	1.942	1.683	2.048
4	7.17	0.1010	0.02450	3.374	2.792	2.270	3.458
5	6.47	0.0912	0.03289	5.174	4.592	2.782	5.053
6	4.97	0.0700	0.03747	5.958	5.356	4.999	13.003
7	4.47	0.0630	0.04106	6.465	5.878	6.227	18.727
8	3.97	0.0559	0.05530	8.699	8.117	8.045	28.126
9	3.57	0.0503	0.06000	9.439	8.857	10.058	40.104
10	3.17	0.0446	0.08719	13.816	13.234	12.895	59.764
11	2.97	0.0418	0.09429	14.833	14.251	14.770	75.597
12	1.77	0.0249	0.29137	45.839	45.257	42.979	389.270

TABLE NO. 1.7

Specifications -	Beam dimensions	1003 X 49.2 X 6 mm
Damping fluid -	Usha Machine Oil	Temperature = 30°C
Viscosity of oil	= 0.24847 N - S / m <sup>2</sup>	
Mass equivalent of the system	= 3.589 Kg	
Natural frequency of the system	= 94.89 rad / S	
Experimental correction for C	= 0.582 N - S / m	
Cylinder diameter	= 70.94 mm    Piston height=12mm	

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp</sub> N - S /m	C <sub>the (1.8)</sub> N - S/m
1	10.67	0.1504	0.01696	1.975	1.393	0.943	0.357
2	9.22	0.1299	0.01389	1.618	1.036	1.306	1.297
3	8.22	0.1158	0.02300	2.679	2.097	1.683	2.048
4	7.17	0.1010	0.03290	3.832	3.250	2.270	3.458
5	6.47	0.0912	0.04181	4.871	4.289	2.782	5.053
6	5.47	0.0771	0.04415	5.143	4.561	4.074	9.260
7	4.97	0.0700	0.04870	5.673	5.091	4.999	13.003
8	3.97	0.0559	0.08120	9.459	8.877	8.045	28.126
9	3.57	0.0503	0.09008	10.494	9.912	10.058	40.104
10	3.17	0.0446	0.13089	15.248	14.666	12.895	59.764
11	2.97	0.0418	0.15209	17.718	17.136	14.770	75.597
12	1.77	0.0249	0.34355	40.023	39.441	42.979	389.272

TABLE NO. 1.8

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Damping fluid - SAE 10

Temperature = 25°C

Viscosity of the oil = 0.5011 N - S / m<sup>2</sup>

Mass equivalent of the system = 3.859 Kg.

Natural frequency of the system = 94.85 rad / S

Experimental correction for C = 0.582 N - S / m

Cylinder diameter = 50 mm Piston height=5mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S /m	C <sub>emp</sub> N - S/m	C <sub>theoretical</sub> N - S/m
1	4.0	0.080	0.03048	3.550	2.968	3.232	6.836
2	3.0	0.060	0.04394	5.117	4.535	5.837	18.632
3	2.6	0.052	0.06390	7.441	6.859	7.891	30.211
4	2.2	0.044	0.07615	8.868	8.286	11.191	52.590
5	1.8	0.036	0.13375	15.576	14.994	16.978	101.162
6	1.4	0.028	0.24873	28.967	28.385	28.532	226.323
7	1.0	0.020	0.51570	60.058	59.476	56.760	653.147
8	0.6	0.012	1.36660	159.153	158.571	160.179	3177.558

TABLE NO. 1.9

Specifications - Beam dimensions 1003 X 49.2 X 6 mm  
 Damping fluid - SAE 10 Temperature = 25°C  
 Viscosity of oil = 0.50111 N - S / m<sup>2</sup>  
 Mass equivalent of the system = 10.7116 Kg.  
 Natural frequency of the system = 56.93 rad / S  
 Experimental correction for C = 1.0246 N - S / m  
 Cylinder diameter = 50 mm Piston height=5mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp.</sub> N - S/m	C <sub>emp.</sub> N - S/m	C <sub>the(48)</sub> N - S/m
1	4.0	0.080	0.01788	3.471	2.946	3.232	6.836
2	3.0	0.060	0.02616	5.077	4.053	5.837	18.632
3	2.6	0.052	0.02807	5.449	4.471	7.891	30.211
4	2.2	0.044	0.04535	8.800	7.775	11.191	52.590
5	1.8	0.036	0.08506	16.504	15.480	16.978	101.162
6	1.4	0.028	0.13965	27.098	26.073	28.532	226.323
7	1.0	0.020	0.52669	102.197	101.177	56.760	653.147
8	0.6	0.012	0.93021	180.497	179.472	160.179	3177.558

TABLE NO. 1.10

Specifications - Beam dimension 1003 X 49.2 X 6 mm  
 Damping fluid - SAE 10 Temperature = 25°C  
 Viscosity of oil = 0.50111 N - S / m<sup>2</sup>  
 Mass equivalent of the system = 10.711 Kg.  
 Natural frequency of the system = 56.93 rad / S  
 Experimental correction for C = 1.0246 N - S / m  
 Cylinder diameter = 50 mm Piston height=12mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	$\delta$	C <sub><math>\delta</math></sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp</sub> N - S/m	C <sub>the</sub> N - S/m
1	3.0	0.060	0.04081	7.918	6.893	14.010	44.718
2	2.6	0.052	0.05421	10.518	9.493	18.938	72.507
3	2.2	0.044	0.07375	14.315	13.290	26.859	126.216
4	1.8	0.036	0.16120	31.290	30.265	40.748	242.784
5	1.4	0.028	0.30542	59.284	58.259	68.477	543.175
6	1.0	0.020	0.61007	118.420	117.325	136.224	1567.550

TABLE NO. 1.11

Specifications - Beam dimension 1003 X 49.2 X 6 mm  
 Damping fluid - SAE 10 Temperature = 25°C  
 Viscosity of oil = 0.50111 N - S / m<sup>2</sup>  
 Mass equivalent of the system = 3.852 Kg.  
 Natural frequency of the system = 94.84 rad / S  
 Experimental correction for C = 0.582 N - S / m  
 Cylinder diameter = 50 mm Piston height = 12mm

S.No.	C <sub>r</sub> mm	C <sub>r</sub> /D <sub>c</sub>	δ	C <sub>δ</sub> N - S/m	C <sub>exp</sub> N - S/m	C <sub>emp.</sub> N - S/m	C <sub>theor.</sub> N - S/m
1	3.0	0.060	0.06147	7.161	6.579	14.010	44.718
2	2.6	0.052	0.08663	10.092	9.510	18.938	72.507
3	2.2	0.044	0.14207	16.551	15.969	26.859	126.216
4	1.8	0.036	0.26648	31.045	30.463	40.748	242.784
5	1.4	0.028	0.45190	52.646	52.064	68.477	543.175
6	1.0	0.020	0.85730	100.081	99.499	136.224	1567.550



TABLE NO. 2.1

Specifications - Beam dimensions 1003 X 49.2 X 6 mm  
 Temperature = 21.5°C  
 Damping fluid - Usha Machine Oil  
 Cylinder diameter = 70.94 mm  
 Piston diameter = 65 mm  
 Viscosity of oil = 0.56294 N-s/m<sup>2</sup>  
 Mass of equivalent system = 10.716 Kg.  
 Natural frequency of system = 56.93 rad/s

S.No.	H mm	$\delta$	C <sub>exp</sub> N - s / m	C <sub>emp</sub> N - s / m
1	5.00	0.08294	15.094	13.91
2	10.00	0.09212	17.747	27.82
3	15.00	0.13686	26.381	41.73
4	20.00	0.17608	33.950	55.64
5	30.00	0.21226	41.558	83.45
6	45.00	0.34000	66.548	125.20
7	55.00	0.41775	82.909	152.99

TABLE NO. 2.2

Specifications - Beam dimension 1003 X 49.2 X 6 mm

Temperature = 25°C

Damping fluid - SAE 10

Cylinder diameter = 50 mm

Viscosity of oil = 0.5011 N-S/m<sup>2</sup>

Mass equivalent of system = 3.859 Kg.

Natural frequency of the system = 94.8+3 rad/s

Piston diameter = 47.2 mm  $C_r = 1.4$  mm

S.No.	H mm	$\delta$	$C_{exp}$ N - S / m	$C_{emp}$ N - S / m
1	5	0.24873	28.96	28.53
2	8	0.32648	38.02	45.64
3	12	0.45190	52.64	68.47
4	17	0.60970	71.01	97.00
5	20	0.70670	82.30	114.12

Piston diameter = 48 mm  $C_r = 1.0$  mm

1	5	0.51570	60.05	56.70
2	8	0.64540	75.16	90.72
3	12	0.85750	100.08	136.22
4	17	1.12000	130.90	192.78
5	20	1.24000	145.40	226.80

TABLE NO. 2.3

Specifications- Beam dimension 1003 X 49.2 X 6 mm

Temperature = 26°C

Damping fluid - SAE 10

Cylinder diameter = 50 mm

Piston diameter = 46.4 mm

Viscosity of the oil = 0.562 N-S/m<sup>2</sup>

Mass equivalent of the system = 3.859 Kg.

Natural frequency of the system = 94.83 rad/s

$$C_r = 1.8 \text{ mm}$$

S. No.	H	$\delta$	$C_{exp}$	$C_{emp}$
	mm		N - S / m	N - S / m
1	5	0.13375	15.57	16.97
2	8	0.20770	24.18	27.15
3	12	0.26648	31.06	40.75
4	17	0.37800	44.08	57.70
5	20	0.46150	53.81	67.90

TABLE NO. 3.1

Specifications- Beam dimensions 1003 X 49.2 X 6 mm

Temperature = 25.3°C

Cylinder diameter = 70.94 mm

Piston diameter = 65.8 mm

Piston height = 8.00 mm

Mass equivalent of the system = 10.712 Kg.

Natural frequency of the system = 56.93 rad/s

$$C_T = 2.57 \text{ mm}$$

S.No.	Name of oil	$\mu$ N-S/m <sup>2</sup>	$\delta$	$\frac{(C_{exp})_{df}}{(C_{exp})_{water}}$	$\frac{(C_{the.})_{df}}{(C_{the.})_{water}}$
1	Air	0.0199	0.00706	0.240	0.199
2	Water	0.1000	0.02946	1.000	1.000
3	SAE 10	0.3854	0.10885	3.694	3.854
4	SAE 40	3.1940	0.79915	27.102	31.940
5	Silicon	1.2665	0.29695	10.077	12.665
6	Usha Oil	2.2803	0.07319	2.484	2.803

TABLE NO. 3.2

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Temperature	= 19.5°C
Cylinder diameter	= 50 mm
Piston diameter	= 45.6 mm
Piston height	= 12.00 mm
Mass equivalent of the system	= 10.7116 Kg.
Natural frequency of the system	= 56.23 rad/s

$$C_r = 2.2 \text{ mm}$$

S.No.	Name of oil	Viscosity N-S/m	$\delta$	$\frac{(C_{exp})_{df}}{(C_{exp})_{water}}$	$\frac{(C_{the})_{df}}{(C_{the})_{water}}$
1	Air	0.0018	0.00394	0.202	0.199
2	Water	0.1000	0.01954	1.000	1.000
3	SAE 10	0.6716	0.09942	5.112	6.716
4	Usha	0.3345	0.05828	3.001	3.345
5	Silicon	1.2460	0.22396	11.500	12.460
6	SAE 30	3.3430	0.59100	30.194	33.430
7	SAE 40	4.9350	0.74900	38.331	49.300

TABLE NO. 3.3

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Temperature = 22°C

Cylinder diameter = 50.00 mm

Piston diameter = 47.2 mm

Piston height = 12 mm

Mass equivalent of the system = 10.7116 Kg.

Natural frequency of the system = 56.93 rad/s

$$C_r = 1.4 \text{ mm}$$

S.No	Name of oil	Viscosity N - S/m	$\delta$	$\frac{(C_{exp})_{df}}{(C_{exp})_{water}}$	$\frac{(C_{the.})_{dt}}{(C_{the.})_{water}}$
1	Air	0.0018	0.01006	0.141	0.150
2	Water	0.1000	0.07140	1.000	1.000
3	Usha	0.3100	0.26500	3.711	3.100
4	SAE 10	0.5620	0.35475	4.967	5.620
5	SAE 30	2.3200	1.39960	19.600	23.200
6	Silicon	1.3560	0.79570	11.144	13.560

TABLE NO. 3.4

Specifications- Beam dimensions 1003 X 49.2 X 6 mm

Temperature = 22°C  
 Cylinder diameter = 50.00 mm  
 Piston diameter = 44.00 mm  
 Piston height = 12.00 mm  
 Mass equivalent of the system = 10.7116 Kg.  
 Natural frequency of the system = 56.93 rad/s

$$C_r = 3.00 \text{ mm}$$

S.No	Name of oil	Viscosity IN - S/m	$\delta$	(C <sub>exp</sub> ) <sub>df</sub>	(C <sub>the</sub> ) <sub>df</sub>
				(C <sub>exp</sub> ) <sub>water</sub>	(C <sub>the</sub> ) <sub>water</sub>
1	Air	0.01832	0.00757	0.757	0.183
2	Water	0.10000	0.01000	1.000	1.000
3	Usha Oil	0.32000	0.03250	3.250	3.200
4	SAE 10	0.56200	0.04885	4.88	5.620
5	SAE 30	2.33000	0.16580	1.650	2.330
6	Silicon	1.25600	0.08785	8.780	12.560
7	SAE 40	4.18000	0.25810	25.800	41.820

TABLE NO. 4.1

Specifications - Beam dimensions 1003 X 49.2 X 6 mm

Temperature = 26°C

Damping fluid - SAE 10, Viscosity of oil = 0.446 N-S/m<sup>2</sup>

Cylinder diameter = 70.94 mm

Piston diameter = 65.00 mm

Piston height = 12.00 mm

$C_r$  = 2.97 mm

Mass equivalent of the system = 10.72 Kg.

Natural frequency of the system = 56.4 rad/S

S.No.	e mm	e/ $C_r$	$\delta$	$\frac{(C_{exp})_e}{(C_{exp})_o}$	$\frac{(C_{the})_e}{(C_{the})_o}$
1	2.47	0.83	0.4682	0.833	0.743
2	1.97	0.66	0.5230	0.930	0.820
3	1.47	0.49	0.5659	1.016	0.891
4	0.97	0.33	0.5693	1.013	0.950
5	0.47	0.16	0.5647	1.005	0.988
6	0.00	0.00	0.5623	1.000	1.000



TABLE NO. 4.2

Specifications - Beam dimensions	1003 X 49 2 X 6 mm
Temperature	= 22°C
Damping fluid - SAE 10, Viscosity of oil	= 0.565 N-S/m <sup>2</sup>
Cylinder diameter	= 50.00 mm
Piston diameter	= 47.2 mm
Piston height	= 12 mm
$C_r$	= 1.4 mm
Mass equivalent of the system	= 17.589 Kg.
Natural frequency of the system	= 46.6 rad/S

S.No.	e mm	e/C <sub>r</sub>	$\delta$	$\frac{(C_{exp})_e}{(C_{exp})_o}$	$\frac{(C_{the})_e}{(C_{the})_o}$
1	1.235	0.882	0.1813	0.637	0.723
2	0.905	0.646	0.2185	0.774	0.819
3	0.575	0.411	0.2515	0.890	0.908
4	0.245	0.175	0.2741	0.971	0.976
5	0.080	0.057	0.2824	1.000	0.991

TABLE NO. 4.3

Specifications - Beam dimensions	1003 X 49.2 X 6 mm
Temperature	= 22°C
Damping fluid - SAE 10, Viscosity of oil	0.565 N-S/m <sup>2</sup>
Cylinder diameter	= 50.00 mm
Piston diameter	= 44.00 mm
Piston height	= 12.00 mm
$C_r$	= 3.00 mm
Mass equivalent of the system	= 10.72 Kg.
Natural frequency of the system	= 56.4 rad/S

S.No.	e mm	e/C <sub>r</sub>	$\delta^*6$	$\frac{(C_{exp})_e}{(C_{exp})_o}$	$\frac{(C_{the})_e}{(C_{the})_o}$
1	2.87	0.956	0.1570	0.564	0.686
2	2.34	0.780	0.1522	0.545	0.766
3	2.01	0.670	0.1664	0.984	0.816
4	1.68	0.560	0.2469	0.884	0.865
5	1.02	0.340	0.1989	0.712	0.945
6	0.69	0.230	0.2383	0.853	0.977
7	0.36	0.120	0.2363	0.846	0.992
8	0.03	0.010	0.2791	1.000	1.000

## CHAPTER - 5

### DISCUSSION OF RESULTS

#### 5.1 CONCENTRIC CASE

Coefficient of damping as given by Peterson (8) is

$$C = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2C_r}{C_r} \right)^3 \quad (18)$$

Thus for a given fluid and a piston height  $C$  is a cubic function of the ratio of the cylinder diameter to the radial clearance.

Large number of experiments were done to verify this theory by changing various parameters such as viscosity, piston height, cylinder diameter and radial clearances. As a further check on the experiments the critical damping of the system was varied. Theoretical and experimental results for these experiments are given in Tables 1.1 to 1.11, 2.1 to 2.3 and 3.1 to 3.4 and also in corresponding Figures.

We note from Eqn. (18) that coefficient of damping depends linearly upon the height of piston and viscosity

of damping fluid. Fig. 2.1 to 2.2 and Fig. 3.1 to 3.4 show that this is observed experimentally.

Dimensional analysis suggests that

$$\frac{C}{\mu H} = f \left( \frac{C_r}{D_c}, \frac{C_r}{H} \right) \quad (37)$$

Originally the experiment was conceived as a study of the dependence of  $C$  on ratio of radial clearance to cylinder diameter as implied by Eqn. (18) above. After data taking was complete, it was realised that the group  $C_r/H$  was also important. Since this parameter is not as small as required by the theory leading to the Eqn. (18), but was of the order of unity in many of the experimental runs. Hence two consolidated plots were attempted that is  $C/\mu H$  as a function of  $C_r/D_c$  and of  $C_r/H$  respectively. These plots revealed that over the ranges of  $C_r/D_c$  and  $C_r/H$  of the present experiments that is 0.012 to 0.1898 and 0.083 to 1.027 respectively,  $C/\mu H$  is primarily a function of  $C_r/D_c$ . Hence only one plot, Fig (6) is presented. On this figure prediction of Eqn. (18) is also plotted. It is observed that there is a considerable difference between the theoretical and experimental values. Disagreement with theory is not unexpected since theory demands  $C_r/H$  to be small where as we note, above, that  $C_r/H$  was of order unity for some runs. However, the disagreement would be

expected to be smaller for smaller values of  $C_r$  (which imply smaller values of  $C_r/H$ ) than for larger values, whereas reverse is observed.

Possible reasons for experimental value of damping coefficient being much smaller than the theoretical ones are:

1. The presence of eccentricity
2. The lack of parallelism between cylinder and piston axes
3. The possibility of an increase in experimental error for low values of  $C_r/D_c$
4. Cavity formation.

The importance of 1 and 2 does indeed increase as  $C_r/D_c$  decreases, however as Eqn. (30) shows, even for maximum eccentricity, the reduction is only about one third of its value. Whereas the disagreement between theoretical and experimental results does not appear to reach any constant ratio.

Of course the experimental error is more for small  $C_r/D_c$  but for smallest  $C_r/D_c$  even, it is only 2.9% (see Appendix-D)

Cavity formation during upstroke is possible since the oil is always at atmospheric pressure on the upper side of the piston. This cavity would collapse on

the down stroke and the reduction in  $C$  would then be large and would systematically increase as  $C_r/D_c$  was decreased. However, the following estimate suggests that cavity formation should not have occurred.

Estimate of Minimum Oil Pressure:

The estimated maximum beam  
deflection was

$$= \pm 2.5 \text{ mm}$$

Maximum frequency in  
any run

$$= 140 \text{ rad/sec.}$$

Therefore maximum piston  
speed

$$= 350 \text{ mm/sec.}$$

Maximum experimental  
value of  $C$

$$= 160 \text{ N - S/m}$$

Therefore maximum force on  
the piston

$$= \frac{160 \times 350}{1000} = 56 \text{ N}$$

Minimum piston area

$$= \frac{\pi}{4} 50^2 \times 10^{-6} \text{ m}^2 = 0.00196 \text{ m}^2$$

Therefore maximum pressure  
difference

$$= \frac{56}{0.00196} = 0.285 \times 10^5 \text{ N/m}^2$$

This is about 0.28 atmospheres, which is a very small value. The vapour pressure of an equivalent oil at room temperature is  $\text{N/m}^2$ , which indicates that cavity

formation should not have occurred in the above series of experiments.

No explanation is thus put forward to explain the disagreement described above. It is left for future investigations to elucidate this.

## 5.2 EMPIRICAL FIT :

It was found that

$$\frac{C}{\mu H} = 3 \pi \left( \frac{D_c - 2C_r}{C_r} \right)^2 \quad (38)$$

fits nearly all the experimental points reasonably well, as seen from Fig. 6 where this relation is also plotted. Eqn. (38) may be found useful to the dash-pot designers.

## 5.3 ECCENTRIC CASE :

In case piston is travelling eccentrically coefficient of damping is given by Eqn. (30),

$$C_e = \frac{3}{4} \pi \mu H \left( \frac{D_c - 2C_r}{C_r} \right)^3 / \left( 1 + \frac{1}{2} \left( \frac{e}{C_r} \right)^2 \right) \quad (30)$$

Theory of Peterson (8) does not give the value of  $C_e$  as a function of eccentricity but it gives its value only for the extreme case of eccentricity to be 40% of that of the concentric case. Eqn. (30) gives the value of  $C_e$  as a function of eccentricity,  $e$ . Value of  $C_e$  decreases

with the increase in  $e$  and for the case of extreme eccentricity value of  $C_e$  is 66.7% of the value of that of concentric case. This difference between the extended theory and Ref. (8) is due to the fact that whereas Ref. (8) obtains  $u_{avg, \theta}$ , the local average velocity, from the expression for  $u_{avg}$  for the concentric case, in the present analysis  $u_{avg, \theta}$  is found from the general solution for  $u(y)$ , Eqn. (11), satisfying the local boundary conditions and then averaging it over the local gap width.

Experimental and theoretical results are given in Tables 4.1 to 4.3 and Figs. 4.1 to 4.3. Experimental results confirm that value of  $C_e$  decreases with increase of  $e$  in the predicted fashion.

In case of large base clearance the experimental points are scattered, see Fig. 4.3. This may be attributed to experimental errors as the value of  $C$  itself is very small to start with. These Figs. still show that  $C_e$  decreases with increase in  $e$ .

Eqn. (30) may also be used to control the coefficient of damping.



5.4 CONCLUSIONS:

(1,8)

1. It has been seen that theory (does not give satisfactory results and demands more thorough understanding of the mechanism
2. Within the range of the present work  $C/\mu H$  was found to be independent of  $C_r/H$  and a function of  $C_r/D_c$  only as suggested by elementary theory, even though this theory is not applicable for  $C_r/H$  of order unity.
3. Empirical formula, given, may be used by engineers to design dash-pots with relatively large clearance within the range of parameters of the present work.
4. Value of coefficient of damping decreases with increase in eccentricity, and is reduced by one third for extreme eccentricity. The empirical formula also caters for the eccentric case.

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APPENDIX-ADETERMINATION OF  $\frac{dp}{dx}$  FOR ECCENTRIC CASE

$dp/dx$  can be determined from fluid flow balance equation (28). Integrating the Eqn. (28) with respect to  $y$ :

$$\frac{\pi}{4} D_p^2 U = \frac{D_p + D_c}{2} C_r \int_0^\pi \left[ -\frac{1}{12\mu} \frac{dp}{dx} (C_r + e \cos \theta)^2 - \frac{U}{2} \right] d\theta \quad (A-1)$$

$$\text{Let } C_r + e \cos \theta = n \quad (A-2)$$

$$\text{therefore } d\theta = -\frac{dn}{e \sin \theta} \quad (A-3)$$

$$\left. \begin{aligned} \text{and } \cos \theta &= \frac{n - C_r}{e} \\ \sin \theta &= \frac{1}{e} \sqrt{e^2 - n^2 - C_r^2 + 2 n C_r} \end{aligned} \right] \quad (A-4)$$

Substituting the values of  $\sin \theta$ ,  $\cos \theta$  and  $d\theta$  in Eqn (A-1)

$$\begin{aligned} \frac{\pi}{4} D_p^2 U &= \frac{D_p + D_c}{2} C_r \int \frac{C_r - e}{C_r + e} \frac{1}{12\mu} \frac{dp}{dx} \frac{n^2 dn}{\sqrt{e^2 - n^2 - C_r^2 + 2 n C_r}} \\ &\quad - \frac{\pi (D_p + D_c)}{4} C_r U \end{aligned}$$

therefore,

$$\frac{dp}{dx} \int \frac{C_r - e}{C_r + e} \frac{n^2 dn}{\sqrt{e^2 - n^2 - C_r^2 + 2 n C_r}} = \frac{24\mu}{(D_p + D_c) C_r} \left[ \frac{\pi}{4} D_p^2 + \frac{\pi}{4} (D_p + D_c) C_r \right] \quad \dots (A-5)$$

Integrating left hand side Ref. (7)

$$\frac{dp}{dx} \left[ \left( -\frac{n}{2} - \frac{6C_r}{4} \sqrt{e^2 - n^2 - C_r^2 + 2n C_r} \right) \right]_{C_{r+e}}^{C_{r-e}} \\ \frac{12 C_r^2 + 4 (e^2 - C_r^2)}{8} \int_{C_{r+e}}^{C_{r-e}} \frac{dn}{\sqrt{e^2 - n^2 - C_r^2 + 2n C_r}} \quad \text{..... (A-6)}$$

Again integrating (C-7) Ref. (7)

$$\frac{dp}{dx} \frac{12 C_r^2 + 4 (e^2 - C_r^2)}{8} \left[ \sin^{-1} \frac{-2n + 2 C_r}{\sqrt{4 C_r^2 + 4 (e^2 - C_r^2)}} \right]_{C_{r+e}}^{C_{r-e}} \quad \text{..... (A-7)}$$

Plugging Eqn. (A-7) in Eqn. (A-5)  $\frac{dp}{dx}$  can be determined as follows

$$\frac{dp}{dx} = - \frac{12 \mu \left[ D_p^2 + (D_p + D_c) C_r \right] U}{(D_p + D_c) C_r (2 C_r^2 + c^2)} \quad \text{(A-8)}$$

APPENDIX B

## VISCOSITY MEASUREMENT

Viscosity of the damping fluid was measured with the help of viscosity meter (14) using the following formula

$$\mu = \frac{N K}{4\pi h \omega} \quad g \quad (B-1)$$

where  $N$  = wire constant

$$K = \frac{1}{r_1^2} - \frac{1}{r_2^2}$$

where  $r_1$  and  $r_2$  are radii of outer radius of plunger and inner radius of smaller cup, respectively.  $r_1 = 0.5$  cms and  $r_2 = 1.5$  cms.

$h$  = depth of immersion

$\omega$  = speed of revolution of cup in rad/S. =  $\frac{2\pi n}{T}$

$n$  = number of revolutions in TSec.

$g$  = acceleration due to gravity.

# APPENDIX C

## SAMPLE CALCULATION

Refer to experimental results Table No. 1.1.

- (a) Calculation for mass equivalent and natural frequency of the system

$$\omega_n = \sqrt{\frac{192 EI}{L^3 M_{eq}}}$$

where

$$E = 2.1 \times 10^{11} \text{ N/m}^2$$

$$I = \frac{1}{12} b d^3 = 0.08856 \times 10^{-8} \text{ m}^4$$

$$L = 1.003 \text{ m}$$

$$M_{eq} = \frac{192}{504} m + M$$

$$m = 2.291 \text{ Kg/m}$$

$$M = 2.983 \text{ Kg.}$$

$$\therefore M_{eq} = \frac{192}{504} \times 2.291 + 2.983$$

$$M_{eq} = 3.859 \text{ Kg.}$$

and

$$\omega_n = \left[ \frac{192 \times 2.1 \times 10^{11} \times 0.08856 \times 10^{-8}}{(1.003)^3 \times 3.859} \right]^{\frac{1}{2}}$$

$$\omega_n = 95.0 \text{ rad/s}$$

(b) Calculation for viscosity determination at 30.5°C

$$\mu = \frac{N K}{4\pi h} \frac{1}{\omega}$$

where

$$K = \frac{1}{(0.5)^2} - \frac{1}{(1.5)^2}$$

$$K = 3.5556 \text{ cm}^{-2}$$

$$N = 0.0810 \times g^6 \text{ cm}^3/\text{M div}$$

$$h = 4.1 \text{ cms}$$

$$\theta = 2^\circ \text{M}$$

$$\omega = \frac{2\pi \times 50}{72}$$

$$= 4.365 \text{ rad/s}$$

$$\mu = \frac{981 \times 0.08109 \times 2 \times 3.5556}{4 \times \pi \times 4.1 \times 4.365}$$

$$= 2.48 \times 10^7 \text{ poise}$$

$$\mu = 0.248 \times 10^7 \text{ N-s/m}^2$$

(c) Experimental value of damping coefficient

Refer to S. No. 13 of Table 1.1

$$C_\delta = \frac{\delta}{2\pi} (2 \text{ Meq } \omega_n)$$

$$= \frac{0.243}{\pi} \times 3.859 \times 95$$

$$C_\delta = 28.299 \text{ N-s/m}$$



## APPENDIX-D

### EXPERIMENTAL ERROR ESTIMATES

Experimental error in the measurement of coefficient of damping may be estimated as follows

$$C_s = \frac{1}{n \pi} \log \frac{x_0}{x_n} \sqrt{\frac{192 E b d^3}{12 L^3}} \text{ Meq} \quad (\text{D-1})$$

Taking logarithm, differentiating partially and making negative sign positive for maximum error, one gets

$$\begin{aligned} \frac{dC_s}{C_s} = & \frac{d(\log x_0)}{\log x_0} + \frac{d(\log x_n)}{\log x_n} + \frac{1}{2} \frac{dE}{E} + \frac{1}{2} \frac{db}{b} + \frac{3}{2} \frac{dd}{d} \\ & + \frac{1}{2} \frac{d \text{ Meq}}{\text{Meq}} + \frac{3}{2} \frac{dL}{L} \end{aligned} \quad (\text{D-2})$$

In the above expression, differential quantities in the numerator are least counts of the quantities measured in the denominator. Here amplitudes  $x_0$  and  $x_n$  were measured with the help of microscope whose least count was 0.01 mm.  $E$  was selected from standard tables in which for mild steel error can be at the most  $0.1 \times 10^{11} \text{ N/m}^2$ . Width of the <sup>beam</sup> was measured with vernier callipers of least count 0.02 mm and height  $d$  of the beam was measured with screw gauge of least count 0.01 mm. Mass attached to beam was weighed on a balance to weigh up to the accuracy of

10 gms. Length of the beam was measured with steel meter scale reading up to the accuracy of 0.25 cms. Now substituting the values of different least counts and average measured quantities in Eqn. (D-2) and multiplying by 100, one gets the estimate of percentage experimental error.

$$\frac{dC_s}{C_s} \times 100 = \left( \frac{0.0009}{2.2925} + \frac{0.0015}{1.7218} + \frac{1}{2} \frac{0.1 \times 10^{11}}{2.1 \times 10^{11}} + \frac{1}{2} \frac{0.02}{49.2} \right. \\ \left. + \frac{3}{2} \frac{0.01}{0.6} + \frac{1}{2} \frac{10}{4000} + \frac{3}{2} \frac{0.25}{100} \right) \times 100 \quad (D-3)$$

$$\therefore \% \text{ error} = 11.7$$

To get each experimental result, system error, damping ratio was subtracted by vibrating the system freely without dash-pot.

And also error in computing theoretical value of coefficient of damping may be estimated as follows

$$\text{Since } C_{\text{the}} = \frac{3}{4} \frac{NK}{4h} \frac{T}{R \times 2 \pi} H^2 \frac{(D_p)^3}{(D_c - D_p)^3} \quad (D-4)$$

Taking the logarithm, differentiating partially and making negative sign positive for maximum error, one gets

$$\frac{dC_{\text{the}}}{C_{\text{the}}} = \frac{dN}{N} + \frac{dK}{K} + \frac{dh}{h} + \frac{dT}{T} + \frac{dH}{H} + 3 \frac{dD_p}{D_p} + 6 \frac{d(D_c - D_p)}{D_c - D_p} \quad \dots \quad (D-5)$$

N and K were read from table Ref. (14) with possible error as indicated below for dN and dK respectively. h was

measured upto the accuracy of 0.5 mm with the help of steel scale. T was measured with accuracy of 2 seconds for 100 seconds.  $H$ ,  $D_p$  and  $D_c$  were measured and the vernier callipers with the least count 0.02 mm. Substituting the different values of least counts and value of average measured quantities in Eqn. (D-5) and multiplying by 100 one gets the estimate of percentage error as follows

$$\frac{dC_{the}}{C_{the}} \times 100 = \left( \frac{0.0005}{0.08109} + \frac{0.05}{3.55} + \frac{0.05}{4.0} + \frac{2}{100} \right. \\ \left. + \frac{0.02}{10} + 3 \cdot \frac{0.02}{50} + 6 \frac{0.01}{0.97 \text{ to } 18.7} \right) \times 100$$

therefore % error = 5.49 and 10.29

For large clearances and small clearances respectively.































